SI Materials and Methods: Testing an alternative (nonlinear) model of NFD

To illustrate the effect of an alternative assumption about the relationship between growth rates and relative abundance, we provide additional results using a non-linear model. All analyses are repeated using the nonlinear fit ($\mathbf{Y}_s = \beta_{0,s} + \beta_{1,s} \log(\mathbf{X}_s) + \boldsymbol{\varepsilon}_s$ for each species s) to estimate NFD. Now, the equilibrium frequency is estimated as $f = \exp(-\beta_{0,s}/\beta_{1,s})$, and frequency dependence (FD) is estimated as the slope of the log-linear relationship, $FD = \beta_{1,s}$. The analyses and results are analogous to those presented in the main text. This analysis does, however, require some minor differences in interpretation. The assumption of a nonlinear form of NFD creates a slightly different threshold for what kind of growth a species can experience and still be considered 'persistent.' As a result, in some communities the number of species classified as persistent differed by one or two relative to the linear analysis. Second, compared to the linear model of NFD, it was less likely for the mean randomized pattern to be negative (Table S4). This may indicate that using a nonlinear model removes some of the biases in the relationship between NFD and frequency (regardless of the appropriateness of the nonlinear assumption for those communities). As a result, more of the communities showing statistically significant ratios of “Observed Covariance” / “Randomized Covariance” take negative values of this ratio (Figure S20-S21).

The assumption of a nonlinear form of NFD changes the results for some individual communities, but does not change the overall conclusion that many ecological communities show a negative relationship between NFD and relative abundance. Using the nonlinear model of NFD changed the conclusion about significance in 30% of communities (see Table S4). This includes 21 communities for which the nonlinear estimates of NFD did not result in a significant

relationship with equilibrium frequency but the linear model did, and 6 communities for which the nonlinear model resulted in a significant relationship where the linear model had not. These results do not tell us, however, for which communities the difference arises because nonlinear NFD is a better assumption, or for which it is a worse assumption. This further supports the need for future exploration into what models provide a superior estimation of NFD and how the shape of this relationship may vary across communities or species.

Table S4: Community-level results from the alternative nonlinear model

"Observed S" is the number of species provided in the raw data. Species included in a community matrix with no or few non-zero abundances, for which estimation of population parameters was obviously not possible, were removed from the dataset before analysis and are not included in the "Observed S". "Persistent S" is the number of species determined persistent by the methods described in the text. "Effect Size" is the proportional difference in the empirical pattern from random, or "Covariance" / "Randomized Covariance". "Covariance" is the estimated covariance between equilibrium frequency and strength of negative frequency dependence for the community. "Randomized Cov" is the mean covariance between equilibrium frequency and strength of negative frequency dependence found in the randomized data. "P-val" is the proportion of randomized pattern values that are less than or equal to the observed pattern. The final column ("Result Different") indicates whether or not the nonlinear model for NFD ultimately resulted in a different conclusion about the significance of the pattern than the linear model used in the main analysis.

Group	Site	Community	Observed S	Persistent S	Time Series Length	Effect Size	Covariance	Randomized Cov	p-val	Result Different
Birds	Hubbard Brook LTER	Songbirds	25	24	42	-5.94031	-0.38811	0.065335	**0.015534**	No
Birds	White Mountain	Songbirds	23	20	15	-2.74344	-0.28249	0.102967	**0.044848**	No
Birds	Redvers	Waterfowl	13	13	26	-1.47458	-0.33985	0.230471	**0.023439**	No
Birds	Skokholm	Songbirds	16	9	52	-92.1876	-1.09833	0.011914	**0.036705**	No
Birds	Konza LTER	Waterfowl	68	64	29	-1.39886	-0.12572	0.089875	0.191585	No
Birds	Konza LTER	Songbirds	7	3	29	-1.05147	-0.40164	0.381983	0.353946	No

Birds	Texas	Songbirds	14	11	16	-0.73223	-0.40615	0.55467	**0.044022**	No
Birds	Eastern Wood	Songbirds	45	30	31	2.218939	0.00965	0.004349	0.605173	**Yes**
Birds	Pawnee	Songbirds	25	22	9	-0.92365	-0.06567	0.071099	0.582502	No
Birds	Pawnee	Raptors	5	3	9	0.068859	0.036422	0.528935	0.555727	No
Birds	Maine	Songbirds	28	21	11	-1.34472	-0.29943	0.222668	0.421492	No
Birds	NewYork	Songbirds	36	28	11	2.569848	0.684357	0.266302	0.721303	No
Birds	Green Mountains	Songbirds	38	34	11	-0.37474	-0.31458	0.839453	0.200038	**Yes**
Birds	Luquillo LTER	Songbirds	19	16	20	-2.24983	-0.20649	0.091779	0.239456	No
Fish	HinkleyPoint	Flatfish	9	5	17	10.74536	-0.74465	-0.0693	0.190888	**Yes**
Fish	HinkleyPoint	Gadoid Fish	14	8	17	-1.15055	-0.49128	0.426992	**0.046765**	No
Fish	North Sea	Demersal Fish	7	7	24	0.096685	0.103891	1.074534	**0.048352**	No
Fish	North Sea	Flatfish	11	11	24	2.001907	1.225256	0.612044	0.735782	No
Fish	North Sea	Gadoid Fish	9	9	24	1.650291	0.504109	0.305467	0.6815	No
Fish	North Sea	Pelagic Fish	3	3	24	-1.06863	-1.43819	1.345831	**0.048352**	**Yes**
Herps	Luquillo LTER	Anoles	3	3	4	0.346314	0.374785	1.082213	0.436859	No
Herps	Bold Park	Snakes	19	14	7	0.10525	0.05491	0.521708	0.579408	No
Herps	Bold Park	Lizards	7	3	7	0.013854	0.018796	1.356721	0.476094	No
Herps	Cowley County	Snakes	6	4	15	0.26118	0.246557	0.944009	0.632624	No
Herps	Cowley County	Lizards	16	7	15	1.459504	0.531516	0.364176	0.709716	No
Herps	Ora Banda	Snakes	9	5	4	0.504828	0.420958	0.833865	0.70225	No
Herps	Ora Banda	Lizards	42	25	4	3.265687	1.529545	0.468369	0.72599	No
Herps	Fitch Nature Preserve	Snakes	3	3	58	1.262598	0.090426	0.071619	0.63023	No
Herps	CoweetaHydrologic Laboratory	Salamanders	4	4	15	-25.6959	0.179778	-0.007	0.719481	No
Herps	E.S.George Reserve	Turtles	3	2	18	0.074445	0.017758	0.23854	0.442214	No
Herps	Rainbow Bay	Frogs	13	13	16	33.50184	2.459568	0.073416	0.739576	**Yes**
Invertebrates	CA Coastline	Molluscs	8	7	66	0.195914	0.43377	2.214081	0.475	No
Invertebrates	Hubbard Brook LTER	Lepidoptera	5	4	12	-2.05702	-1.83869	0.893863	**0**	No
Invertebrates	Jornada – Creosote	Creosote Insects	113	11	6	0.197888	0.110525	0.558526	0.637318	**Yes**
Invertebrates	Jornada – Grassland	Grassland Insects	132	12	6	6.101269	4.7729	0.78228	0.728275	No

Inverte-brates	Jornada – Mesquite	Mesquite Insects	62	8	6	6.819689	0.532544	0.078089	0.724833	No
Inverte-brates	Jornada – Tarbush	Tarbush Insects	122	13	6	-3.04552	-0.52619	0.172774	**0.096397**	No
Inverte-brates	Luquillo LTER	Snails	18	16	17	-0.4779	-0.12532	0.262235	**0.065018**	No
Inverte-brates	Oneida Lake	Zooplankton	33	24	32	-0.58255	-0.18824	0.323135	0.35513	No
Inverte-brates	Pacific Coast – CAY	Arthropods	12	3	11	0.072552	0.068978	0.950743	0.664432	No
Inverte-brates	Pacific Coast	Echinoderms	8	3	11	-3.31097	0.635764	-0.19202	0.689646	No
Inverte-brates	Pacific Coast – BOA	Molluscs	40	6	11	0.215578	0.466224	2.162668	0.492709	**Yes**
Inverte-brates	Pacific Coast – CAY	Molluscs	40	8	11	0.059595	0.157587	2.644288	0.283436	No
Inverte-brates	Pacific Coast – GPT	Molluscs	40	6	11	0.532328	2.847121	5.34843	0.635364	No
Inverte-brates	Pacific Coast – HAZ	Molluscs	40	5	11	0.279098	1.213137	4.346633	0.352758	No
Inverte-brates	Pacific Coast – MCR	Molluscs	40	6	11	0.567212	2.409846	4.248582	0.58131	No
Inverte-brates	Pacific Coast – OCC	Molluscs	40	6	11	0.512725	2.874503	5.606328	0.544573	No
Inverte-brates	Pacific Coast – PSN	Molluscs	40	5	11	0.438192	1.735571	3.960759	0.556511	No
Inverte-brates	Pacific Coast – SAD	Molluscs	40	6	11	-0.6059	-4.03211	6.65471	**0.035631**	**Yes**
Inverte-brates	Pacific Coast – SHB	Molluscs	40	6	11	0.852494	2.250554	2.639965	0.690737	No
Inverte-brates	UK Nature Reserves - Agricultural	Butterflies	23	21	10	10.78975	4.350889	0.403243	0.73736	**Yes**
Inverte-brates	UK Nature Reserves - Coastal	Butterflies	28	26	10	-4.02547	-0.68702	0.170669	**0.006657**	No
Inverte-brates	UK Nature Reserves - Forest	Butterflies	33	32	10	-2.98659	-0.63206	0.211633	**0.029588**	No
Inverte-brates	UK Nature Reserves - Grassland	Butterflies	36	33	10	-2.47639	-0.36726	0.148306	**0.010652**	No
Inverte-brates	UK Nature Reserves - Mixed	Butterflies	32	29	10	1.81467	0.270688	0.149167	0.720239	**Yes**
Inverte-brates	UK Nature Reserves - Wetlands	Butterflies	25	24	10	-1.13919	-0.98175	0.861791	**0.009986**	No
Mam-mals	Portal	Rodents	9	9	22	6.669527	2.239659	0.335805	0.737712	**Yes**
Mam-mals	Sevilleta LTER - 5pgrass	Rodents	11	4	17	-1.09558	-0.56487	0.515589	**0.094163**	No
Mam-mals	Sevilleta LTER - 5plarrea	Rodents	12	5	17	-1.05931	-0.30379	0.28678	0.194914	**Yes**
Mam-mals	Sevilleta LTER – rslarrea	Rodents	15	10	17	-0.45065	-0.10838	0.24049	0.277552	**Yes**
Mam-mals	Konza LTER	Rodents	7	4	17	-3.25031	-0.73052	0.224755	**0.027245**	No
Mam-mals	Curlew Valley	Rodents	5	4	14	1.150824	0.844352	0.733694	0.676523	No
Mam-mals	INEEL	Rodents	3	3	12	0.632008	1.565532	2.477077	0.538481	No
Mam-mals	Jornada LTER – Grass	Rodents	6	2	13	0.079349	0.067728	0.853548	0.129774	No

Mammals	Jornada LTER – Shrub	Rodents	11	4	13	-0.36076	-0.08325	0.230768	0.432683	No
Mammals	Powdermill	Squirrels	2	2	21	-0.32519	-0.32989	1.014434	**0.059528**	No
Mammals	Powdermill	Rodents	3	3	21	1.675663	0.465213	0.277629	0.676464	No
Mammals	Shortgrass	Rodents	5	4	7	-52.0118	-1.59744	0.030713	**0.074535**	**Yes**
Mammals	Ontario	Rodents	6	6	44	-0.57969	-0.1774	0.306027	**0.077571**	**Yes**
Mammals	Mont St. Hilaire	Rodents	3	3	11	6.918303	8.546077	1.235285	0.724359	**Yes**
Mammals	Karoo National Park	Ungulates	14	10	16	-1.19782	-0.67021	0.559525	**0.04609**	**Yes**
Mammals	Kruger National Park	Ungulates	11	8	17	16.60191	2.910578	0.175316	0.739691	**Yes**
Mammals	Golden Gate National Park	Ungulates	9	5	11	-0.63173	-0.34697	0.549245	0.161624	**Yes**
Plants	Desert Laboratory – Open	Annuals	55	29	29	15.30599	5.756442	0.376091	0.737281	**Yes**
Plants	Desert Laboratory – Shrub	Annuals	54	30	21	0.129661	0.070267	0.541925	0.588362	**Yes**
Plants	Jornada LTER – Creosote	Summer Annuals	111	26	20	0.464771	0.541942	1.166042	0.681788	**Yes**
Plants	Jornada LTER – Creosote	Winter Annuals	83	19	19	0.49692	1.212155	2.439336	0.679119	No
Plants	Jornada LTER – Grassland	Summer Annuals	167	67	20	-0.04442	-0.04793	1.078834	0.372157	No
Plants	Jornada LTER – Grassland	Winter Annuals	119	47	19	0.798224	0.938158	1.175306	0.698112	**Yes**
Plants	Jornada LTER – Mesquite	Summer Annuals	79	18	20	2.023217	3.49885	1.72935	0.725562	No
Plants	Jornada LTER – Mesquite	Winter Annuals	61	12	18	-0.41903	-1.26822	3.026545	**0.039079**	No
Plants	Jornada LTER – Playa	Summer Annuals	84	18	20	0.310785	0.409398	1.317302	0.659628	**Yes**
Plants	Jornada LTER – Playa	Winter Annuals	53	6	19	0.959518	4.025516	4.195351	0.713591	No
Plants	Jornada LTER – Tarbush	Summer Annuals	99	29	20	-1.04318	-1.31311	1.258755	**0.008876**	No
Plants	Jornada LTER – Tarbush	Winter Annuals	81	22	19	0.265679	0.547622	2.061218	0.561484	No
Plants	Kansas mixed-grass prairie	Annuals	34	5	39	0.500958	2.106724	4.205389	0.675389	No
Plants	Kansas mixed-grass prairie	Perennials	102	52	41	38.69472	36.95326	0.954995	0.739112	No
Plants	Portal	Summer Annuals	68	40	32	-0.71949	-0.56391	0.78376	**0.043434**	**Yes**
Plants	Portal	Winter Annuals	71	38	32	0.024081	0.014857	0.616956	0.541248	**Yes**
Plants	Sagebrush Steppe	Annuals	73	10	27	0.450497	0.436405	0.96872	0.69655	**Yes**

Figure Legends

Figure S20: Results were compared to randomized data to account for bias introduced by measurement error and determine which relationships were significant (colored bars) or not (grey bars), using the alternative nonlinear fit ($\log(N_{t+1}/N_t) = b_0+b_1*\log(p_t)$) to estimate NFD (1182 species, 90 communities). P-values were calculated as the proportion of randomized values larger than the observed asymmetry value. We detected a significantly asymmetric NFD pattern in 28% of these communities.

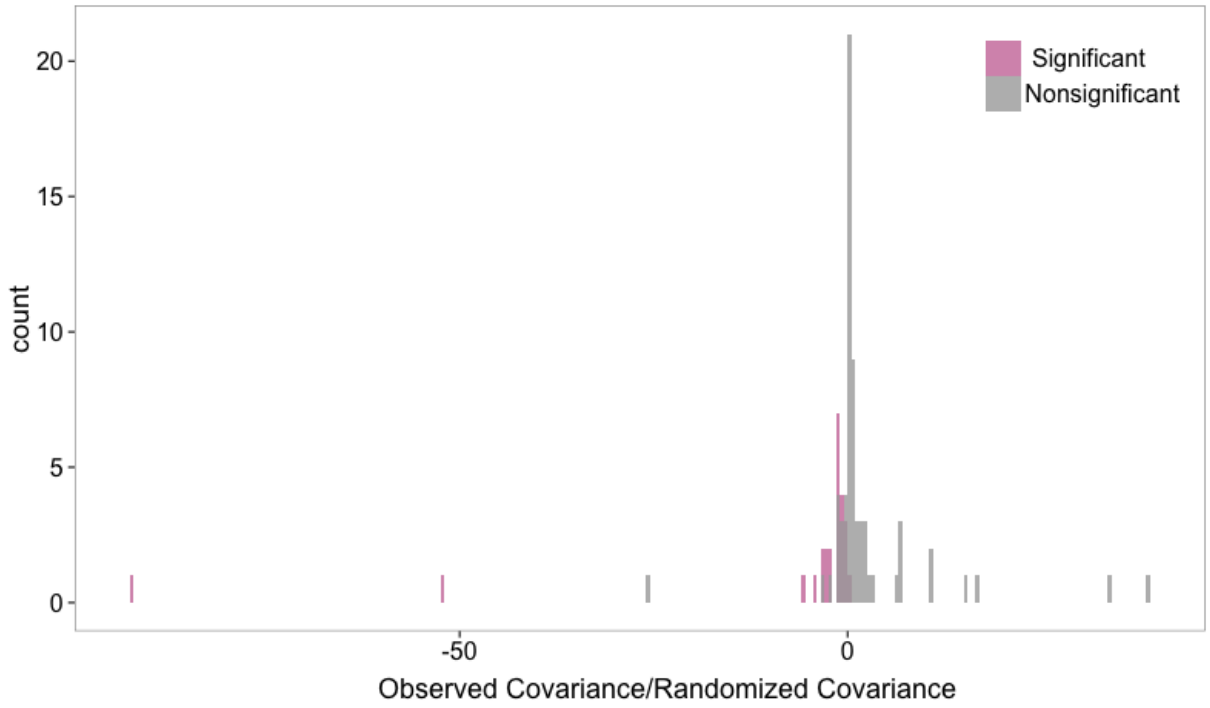

Figure S21: Results separated by taxonomic group, showing the relative size of the asymmetry in NFD between species in each community (Observed Covariance/Randomized Covariance), using the alternative nonlinear fit ($\log(N_{t+1}/N_t) = b_0+b_1*\log(p_t)$) to estimate NFD. The proportion value in each panel is the proportion of communities in that group with a significant asymmetric NFD pattern.